Reading Notes

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There is a theorem that says that every subsequence of a convergent sequence converges to the same limit of the original sequence.

There is another theorem called the **Bolzano-Weirestrass Theorem** that says that every bounded sequence contains a convergent subsequence.

We use these two in unison to show the **Cauchy Criterion**, which is the double implication that a sequence converges *if and only if* it is a Cauchy sequence.

The proof of the **Cauchy Criterion** relies on the above two theorems and a Lemma that says that Cauchy Sequences are bounded. From the Lemma we know that the Cauchy sequence is bounded, and from the **Bolzano-Weirestrass Theorem** we can say that this bounded sequence contains a convergent subsequence, and then from that we can use the first theorem to say that the original sequence converges to the limit of this subsequence.